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Measurement of Aerodynamic Heating of Wind-Tunnel Models by Means of Temperature-Sensitive Paint

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Nomenclature

h = heat-transfer coefficient, Btu/ft^2-sec-°R
M = Mach number
Re = Reynolds number/ft, ft^{-1}
t = time, sec
α = angle of attack, deg
β = angle of yaw, deg
θ = complement of flow deflection angle, deg

Subscripts

∞ = freestream conditions
s = stagnation point
0 = stagnation conditions behind a normal shock in test section on 0.375-in. radius sphere

Introduction

Temperature-sensitive paints have been used for several years to obtain a visualization of the aerodynamic heating distribution on wind-tunnel models. An extensive series on X-20 glider models was done in 1961 in the Arnold Center Von Karman Facility C Tunnel at $M = 10.2$ and $Re = 2.2 \times 10^6$/ft. A four-color-change paint was used. Efforts were undertaken subsequently at The Boeing Company to obtain not only qualitative but also quantitative information from thermal paint tests. A method developed by Lorenz and Sartell[1] was successfully applied to some simple configurations. It is the purpose of this note to illustrate the application of the method to a complicated configuration where the results can be checked by thermocouple measurements.

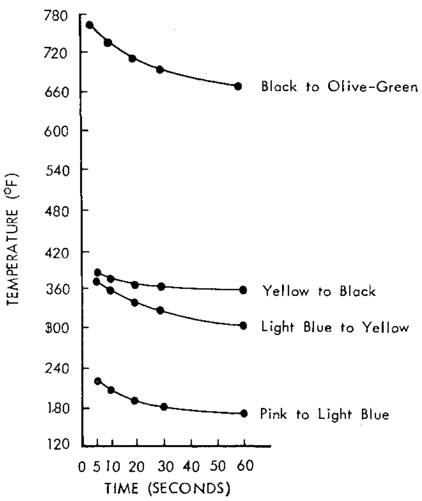

Fig. 1 Exposure required to change color of Detectotemp 915-0933 paint.

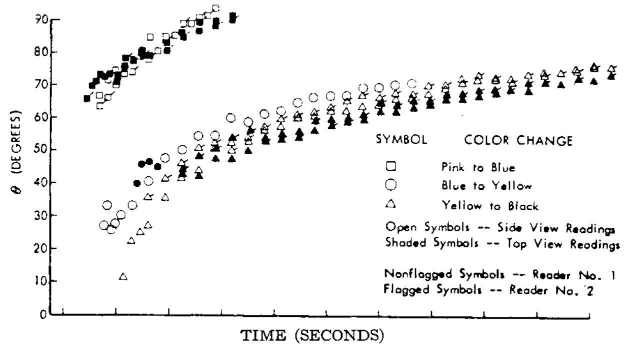

Fig. 2 Six-inch sphere color change history.

Method and Models

The testing procedure is as follows:

1) The model and one or several calibration spheres are painted. A four-color-change paint was selected for this test. The paint calibration diagram (color-change temperature vs exposure time) is shown in Fig. 1.

2) The model and calibration spheres are separately injected into the wind tunnel, and color movies are taken of the color changes.

3) The calibration-sphere film is scanned frame by frame to establish the position θ of the boundaries between colors (e.g., pink to blue) vs time (see upper part of Fig. 2).

4) The θ vs t curves are cross plotted against the theoretical h/h_s vs θ curve for the sphere.[2] This leads to three curves of h/h_0 vs t, or frame number (lower part of Fig. 2). The fourth color change, black to olive, could not be observed. These are the final calibration curves that will be used for measuring h/h_0. — h_0 is a reference quantity defined as the heat-transfer coefficient at the stagnation point of a 0.75-in.-diam sphere under the same flow conditions ($h_0 = 0.032$ Btu/ft^2-sec-°R).

5) The model film is then scanned until that particular frame is found where the first color boundary (e.g., pink to blue) passes through a chosen station. The corresponding value of h/h_0 is then read from Fig. 3.

This procedure is repeated as successive color boundaries pass through the station, so that, in principle, four values of h/h_0 should be read for each station. These values should coincide or be very close. If significant discrepancies exist, they are due to conduction differences between the model and the sphere. In this case the first color change that is

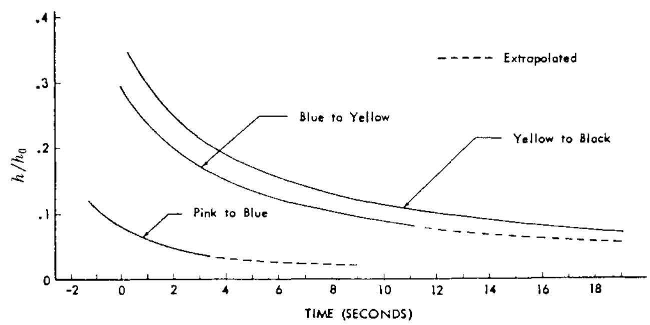

Fig. 3 Heat-transfer calibration curves.

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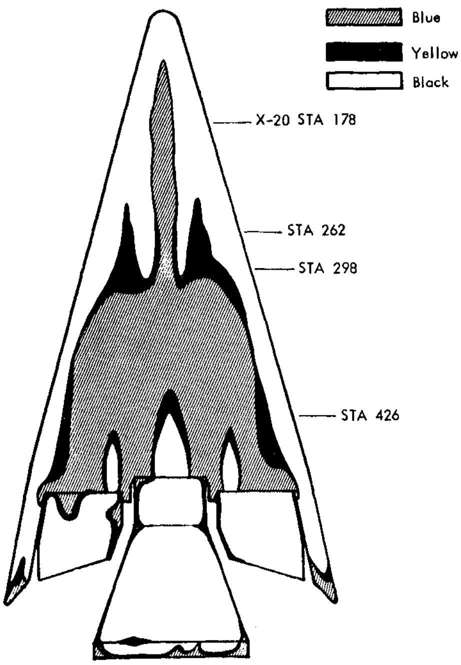

Fig. 4 Typical final color patterns: model lower surface, 15° angle of attack.

observed at the station will give the most reliable value of h/h_0.

The material for the models is Glasrock, a slip-cast, fused silicate, selected because of its low thermal diffusity, ease of forming, extremely low coefficient of thermal expansion, adequate strength, and machinability. The paint used for this test is distributed by the H. V. Hardman Company under the trade name Detectotemp. There are many varieties of this paint, with capabilities of one to four color changes up to 2462°F. Detectotemp 915-0933 was chosen for this test; its color changes (Fig. 1) are essentially permanent. Proper preparation and application of the paint on the model[3] are critical for successful testing. In this test, six $\frac{1}{20}$-scale models of the X-20 glider, a 6-in.-diam sphere, and a 3-in.-diam sphere were tested. Each glider model had deflected control surfaces.

The tests were run in the Arnold Center C Tunnel at $M_\infty = 10.2$, $Re_\infty = 2.2 \times 10^6$/ft, $\alpha = 0°$ to 40°, and $\beta = 0°$ to 10°. The models were mounted on the sting in the cooling chamber beneath the tunnel and then injected into the flow. Care was taken to insure the same flow conditions for the glider models and the calibration spheres. Two movie cameras, one above the model and one on the port side, were used simultaneously to record the color changes. The models were kept in the flow between 18 and 25 sec to achieve all four color changes and were then retracted. A thin protective coat of acrylic lacquer was immediately sprayed on each model. Figure 4 shows a typical color pattern.

The cameras were operated at 128 frames/sec. The films were scanned using a Vanguard Motion Analyzer, which projects the film frames on a ground glass screen. Frames are counted automatically, and stations on the image can be located by means of adjustable cross wires. A considerable amount of calculation was needed to go from the "screen" coordinates of a given station to its physical coordinates on the model. It would be very desirable in future tests to have a coordinate gridwork painted directly on the models. (Some difficulties were also experienced in locating the color boundaries exactly because of the poor resolution of motion pictures. A sequence camera should be more suitable.) The reference frame (time zero) was chosen when the model reached tunnel centerline; this explains the occurrence of negative time values in Fig. 3. Heat-transfer coefficients were measured at those stations on the glider models at which thermocouple measurements had already been taken on geometrically identical models at the same flow conditions.

Results

Plots of h/h_0 vs X/L (or S/L) and thermocouple number are presented in Figs. 4 and 5. Both thermocouple measurements (open circles) and thermal paint measurements (filled circles) are shown at each station. At many stations more than one color change was observed, so that several measurements of h could be made; these are denoted by flagged circles. The agreement between thermocouple and thermal paint measurements was fair to good; in all cases the trends were similar. The agreement on the upper surface is poorer than on the lower surface because the measurements are made at the lower end of the paint's thermal sensitivity. Only one color change (pink to blue) could be observed, and for some stations the calibration curve (Fig. 2) had to be extrapolated to higher t values in order to read h/h_0. A paint that was sensitive in a lower temperature range would have given better results. Conversely, the model could have been kept in the flow for a longer time at the cost of burning the lower surface.

Conclusion

A simple technique has been developed for semiquantitatively measuring heat-transfer coefficients by means of temperature-sensitive paints. The method is particularly suitable for preliminary design and proposal work because of the short construction time for the model, its low cost

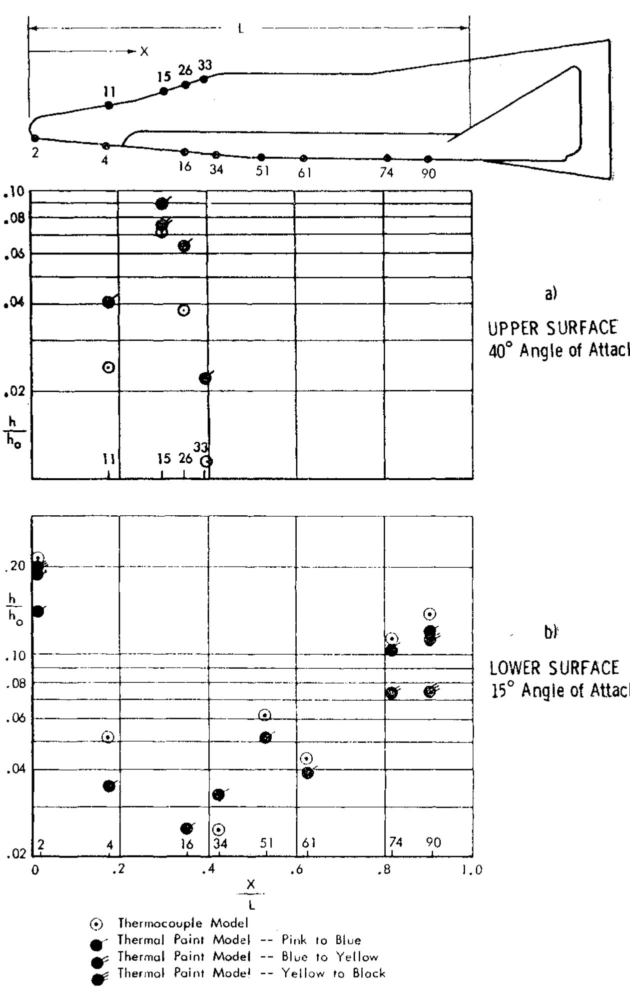

Fig. 5 Centerline heat-transfer distribution.

(one-fifth to one-tenth of an instrumented model), and the ability to obtain isotherms easily. Significant improvements in accuracy could be obtained by using a sequence camera instead of a movie camera and by painting a coordinate network on the models.

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Separation of Satellites in Near-Circular Orbits by Circumferential Impulse

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Nomenclature

a	= semimajor axis
b	= semiminor axis
e	= orbital eccentricity
E	= eccentric anomaly
F	= principal focus of ellipse
M	= mean anomaly
n	= number of orbits subsequent to separation
0	= origin of rectangular coordinate system
$P(x, y)$	= position at time t of satellite in lower energy orbit
P'	= position at time t of satellite in higher energy orbit
r	= central radius
S	= separation distance
t	= time from separation
T	= orbit period
V	= satellite velocity
ΔV	= impulsive velocity increment
β	= view angle (Fig. 2)
η	= true anomaly
θ	= elevation angle (Fig. 2)

Superscript

*	= condition at passage point

A SATELLITE in a central gravitational field is in near-circular orbit. A second satellite is separated from the first by means of a small impulsive velocity increment directed collinearly or anticollinearly with the velocity vector. It is desired to describe the flight-path histories following separation in terms of distance and view angle between satellites.

By the method of differentials, it is found that, to the first order, the lower-energy satellite overtakes that of higher energy when both have traveled through 73.09° of central angle η (Fig. 1). The separation distance S at a given η is related linearly to the central radius r and the nondimensional separation velocity $\Delta V/V$, but the view angle β (the angle between the local horizontal, for the lower-energy satellite and the line connecting the two satellites), is independent of r and $\Delta V/V$.

Analysis

The time from periapsis in an elliptical orbit can be expressed in terms of orbit period T and mean anomaly M

$$t = TM/2\pi \tag{1}$$

Differentiating,

$$dt = (TdM + MdT)/2\pi \qquad M = E - e\sin E \tag{2}$$

where E is the eccentric anomaly (Fig. 1), and e is the orbital eccentricity; to the first order, $dM = dE - de \sin E$, so that

$$dt = [T(dE - de\sin E) + E\,dt]/2\pi \tag{3}$$

Since comparison of elements is to be made at a given time, $dt = 0$; it follows that

$$dE = de\sin E - E\,dt/T \tag{4}$$

The lower-energy satellite initially trails its sister, but by losing altitude it gains velocity and overtakes its sister during the first orbit; a second passage does not occur until many orbits later. At passage, the two satellites have equivalent central angle η, measured about the principal focus from the initial apsidal point (Fig. 1). The central angle thus defined is equivalent to the true anomaly when measured from periapsis, or 180° minus the true anomaly when measured from apoapsis. In the present context, it is mathematically permissible to assume that the problem always initiates at periapsis, η then being equivalent to the true anomaly. The relationship between E and η is known to be [Ref. 1, Eq. (4-113)]

$$\tan(\eta/2) = [(1+e)/(1-e)]^{1/2}\tan(E/2) \approx (1+e)\tan(E/2) \tag{5}$$

Differentiation of (5) gives

$$\tfrac{1}{2}\sec^2(\eta/2)d\eta = \tfrac{1}{2}\sec^2(E/2)dE + de\tan(E/2)$$

But, at the point of passage, $d\eta = 0$, so that

$$dE^* = -[2de\tan(E^*/2)]/[\sec^2(E^*/2)] = -de\sin E^* \tag{6}$$

where the asterisk indicates the initial passage point. Substituting (6) into (4) and solving for E^* gives $E^* = (\frac{4}{3})\sin E^* = 73.09°$. This result confirms the limit case indicated by Milstead [Ref. 2, Eq. (7)]. The separation distance S^* is found next. From Ref. 1, Eq. (4-99),

$$r = a(1 - e\cos E) \tag{7}$$

where a is the semimajor axis of the ellipse. Differentiating this relation,

$$dr/a = (da/a) - de\cos E \approx dr/r \tag{8}$$

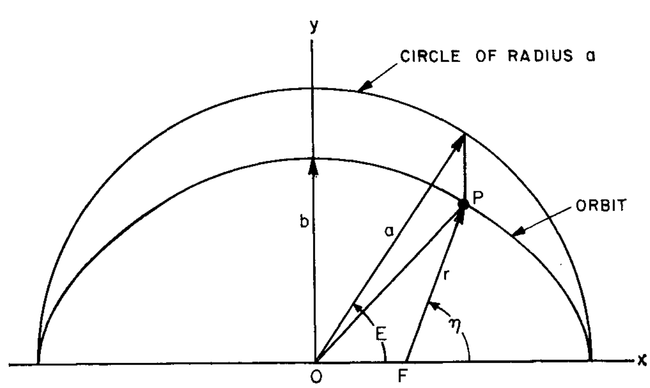

Fig. 1 Orbit notation.

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